EAGER/PSGE - EXCISE PROJECT

METHODOLOGICAL NOTE 2

ESTIMATING DEMAND CURVES FOR GOODS SUBJECT TO EXCISE TAXES

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ABSTRACT

This methodological note summarizes the practical steps required to specify and estimate demand curves for excisable goods such as gasoline, beer and cigarettes. The method is applied to the estimation of the demand for regular gasoline in Madagascar over the period 1978-1996. The note concludes with a brief outline of the main estimation issues, and summarizes the most important empirical results from other studies that estimate the demand for gasoline, tobacco products, and alcoholic beverages.

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1. Introduction

What happens to government revenue when the tax on a good is changed? And what tax rate maximizes government revenue? To answer these questions one needs to know the form of the demand and supply curves for the good. In this methodological note we set out a procedure for estimating demand curves.

For the main commodities subject to excise - notably petroleum products, alcoholic beverages and tobacco products - it is conventional (and usually reasonable) to assume that supply is infinitely elastic, particularly when annual data are being used. This gives the horizontal supply curve as shown by Q_s in Figure 1. When taxes are added, this gives the tax-inclusive supply curve Q_s+T. From year to year the cost of supplying the goods, and/or the tax rate, varies. This moves the tax-inclusive supply curve up and down, tracing out equilibrium points (such as A) along the demand curve. Thus every price and quantity combination which is observed, in each year, must be on the demand curve. The essential idea behind estimating a demand curve is to put numbers on this relationship between price and quantity.

The next section lays out the main methods for estimating demand curves, with the help of a relevant example. For the excise study it would be worth following the first 8 steps outlined in this section. The methodological section is followed by a brief and highly selective summary of the main estimation issues and results for tobacco and alcohol demand.

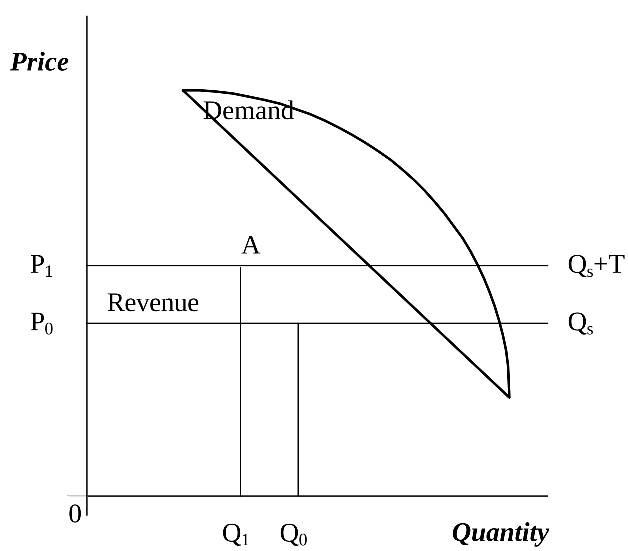

Figure 1

2. The Practical Estimation of Demand Curves

In this section we summarize the main approaches to estimating demand curves. To illustrate the techniques, we apply them to data on the consumption of regular gasoline ("essence tourisme") in Madagascar during the period 1978-1996. The data are reproduced in Appendix 1. The output included in the text was produced using LIMDEP version 7.0, but the estimates could equally well have been done with another software package (e.g. SAS, SPSS, TSP, RATS, Microfit, STATA).

Step 1. Choosing the variables

The first step is to determine what variables are to be included in the analysis. The choice is based both on theory and on practical availability.

The dependent variable in looking at gasoline demand is typically taken to be the amount of gasoline used per capita but some authors prefer to use gasoline used per driver or per vehicle mile driven. In the Madagascar case the lack of good information on the number of drivers or vehicle miles driven rules out the latter two possibilities, so one is obliged to use gasoline consumption per capita. Consumption should be measured as the volume (e.g. liters, tonnes) bought by consumers (including businesses who need gasoline) during the time period in question.

Theory suggests that the independent variables should include:

a. *The price of gasoline.* This should be deflated, for instance by the consumer price index, to give the real price of gasoline; one may think of the real price of gasoline as the price of gasoline relative to the price of all other goods and services. We expect a higher price of gasoline to be associated with a lower quantity of gasoline demanded; this is the demand relationship of primary interest.

b. *The price of substitutes and complements.* The most important substitute for gasoline is diesel fuel. If the real price of diesel fuel rises we expect the quantity of gasoline demanded to rise, as consumers substitute away from diesel fuel. Cars and light trucks and buses, which run on gasoline, are complementary goods, and in principle the price of these vehicles should be included, the idea being that if cars are more expensive then fewer will be bought and the demand for gasoline will be lower (other things being equal). In practice it is extremely difficult to generate a good price series for cars, mainly because the quality of cars changes significantly over time and so it is difficult to obtain the price of a "standardized" car.

c. *Income.* With higher incomes, more individuals and businesses can afford to run vehicles, and so the demand for gasoline will be higher. If the dependent variable is gasoline consumption per capita, then income needs to be expressed in per capita terms too. The most commonly-used variable is real GDP/capita, but one could make a case for using real consumption expenditure per capita or real disposable income per capita instead, if these are available.

Step 2. Build the data set.

This is often the longest and most tedious step. Except with large data sets, it is generally helpful to organize the data on a spreadsheet (such as Excel or Lotus123), and even to use the spreadsheet to do some initial transformations of the data (e.g. converting GDP into real GDP in, say, 1996 prices).

Step 3. Exploratory data analysis.

It is usually worth getting summary statistics (mean, standard deviation, minimum, maximum) for the variables to be used, and even to graph them. For instance:

```
================================================================================
º  LIMDEP Estimation Results                       Run log line   51  Page  23 º
º  Sample was reset:  SAMPLE   ALL                                             º
º  Current sample contains      19 observations.                               º
===============================================================================¼

                                           Descriptive Statistics
  Variable   Mean      Std. Dev.   Skew. Kurt.  Minimum     Maximum      Cases
  ____________________________________________________________________________
  QREGCAP    8065.3864   2161.5976   1.4   3.6   5870.1479  13569.5391      19
  RPREG1        7.2377      1.1445   0.1   2.4      5.3078      9.6510      19
  RPDIE1        3.9178      0.3433   0.2   2.2      3.3037      4.6221      19
  RGDPCAP    1356.3608    149.2982   1.5   3.8   1217.7950   1724.6815      19
```

The variables here are

QREGCAP: volume of gasoline consumption per capita (kg p.a.)
RPREG1: price at the pump of regular gasoline in FMG per liter in 1990 prices.
RPDIE1: price at the pump of diesel gasoline in FMG per liter in 1990 prices.
RGDPCAP: real GDP per capita, in '000 of FMG in 1990 prices.

Step 4. Basic OLS estimations

It is a good idea to begin with the simplest type of regression, which is ordinary least squares (OLS). Here is the result of such an estimation:

```
================================================================================
º  LIMDEP Estimation Results                         Run log line   20  Page   1 º
º  Sample was reset:  SAMPLE   ALL                                              º
º  Current sample contains      19 observations.                                º
================================================================================¼
   ==========================================================================
   º Ordinary    least squares regression    Weighting variable = ONE      º
   º Dependent variable is QREGCAP   Mean = 8065.38638, S.D. =   2161.5976 º
   º Model size: Observations =      19, Parameters =   4, Deg.Fr. =    15 º
   º Residuals:  Sum of squares=   0.190725E+08 Std.Dev. =      1127.60673 º
   º Fit:        R-squared = 0.77323, Adjusted R-squared =         0.72788 º
   º Model test: F[  3,     15] =   17.05,    Prob value =         0.00004 º
   º Diagnostic: Log-L =   -158.2433, Restricted(á=0) Log-L =    -172.3396 º
   º             Amemiya Pr. Crt.=*********, Akaike Info. Crt.=     17.078 º
   º Autocorrel: Durbin-Watson Statistic =   1.28739,   Rho =      0.35631 º
   ==========================================================================¼
     Variable  Coefficient   Standard Error  t-ratio  P[|T|>=t]   Mean of X
     ______________________________________________________________________
      Constant    -12732.          4218.9        -3.018   0.00865    n.a.
       RPREG1     -81.910           241.43       -0.339   0.73911   7.238
       RPDIE1      981.71           806.79        1.217   0.24248   3.918
       RGDPCAP     12.935           1.8203        7.106   0.00000   1356.
```

The fit is relatively good, with an adjusted R^2 of 0.73. The coefficients have the expected signs, although the price variables are not statistically significant. The relatively low Durbin-Watson statistic indicates the presence of autocorrelation, which will need to be addressed. One commonly-used fix is to add a time trend, which gives the following:

```
   ==========================================================================
   º Ordinary    least squares regression    Weighting variable = ONE      º
   º Dependent variable is QREGCAP   Mean = 8065.38638, S.D. =   2161.5976 º
   º Model size: Observations =      19, Parameters =   5, Deg.Fr. =    14 º
   º Residuals:  Sum of squares=   0.637309E+07 Std.Dev. =       674.70054 º
   º Fit:        R-squared = 0.92422, Adjusted R-squared =         0.90257 º
   º Model test: F[  4,     14] =   42.69,    Prob value =         0.00000 º
   º Diagnostic: Log-L =   -147.8298, Restricted(á=0) Log-L =    -172.3396 º
   º             Amemiya Pr. Crt.=*********, Akaike Info. Crt.=     16.087 º
   º Autocorrel: Durbin-Watson Statistic =   2.43274,   Rho =     -0.21637 º
   ==========================================================================¼
     Variable  Coefficient   Standard Error  t-ratio  P[|T|>=t]   Mean of X
     ______________________________________________________________________
       Constant   0.80144E+06     0.15417E+06    5.198   0.00013    n.a.
       RPREG1     -1193.4          255.24       -4.675   0.00036   7.238
       RPDIE1      2300.5          543.49        4.233   0.00084   3.918
       RGDPCAP     3.0062          2.1726        1.384   0.18811   1356.
       YEAR       -401.52          76.021       -5.282   0.00012   1987.
```

The results are not entirely convincing. It is unusual to find a negative time trend in this context. What is probably occurring is that the time trend is picking up some of the effect that should rightfully be attributed to the fall, over time, in real GDP per capita. The clearest evidence of this is the substantial drop in the coefficient on real per capita GDP (RGDPCAP) when the time trend is added to the regression.

Step 5a: Explore functional form

Theory gives very little guidance as to the appropriate functional form for the regression, yet the choice is important when using the econometric results to estimate the effect of changes in the tax rate on the revenue yielded by the tax (see Haughton 1998). While the linear form shown above is sometimes estimated, it is far more common to estimate demand equations in log form - partly because the results are easier to interpret. Such an estimation gives the following result:

```
================================================================================
o  LIMDEP Estimation Results                       Run log line   22  Page   3 o
o  Current sample contains      19 observations.                               o
===============================================================================¼
   ========================================================================
   o Ordinary    least squares regression    Weighting variable = ONE      o
   o Dependent variable is LQREGCAP  Mean =    8.96655, S.D. =      0.2367 o
   o Model size: Observations =      19, Parameters =   4, Deg.Fr. =    15 o
   o Residuals:  Sum of squares=   0.271745     Std.Dev. =         0.13460 o
   o Fit:        R-squared = 0.73054, Adjusted R-squared =         0.67665 o
   o Model test: F[  3,     15] =   13.56,    Prob value =         0.00015 o
   o Diagnostic: Log-L =     13.3898, Restricted(á=0) Log-L =       0.9322 o
   o             Amemiya Pr. Crt.=    0.022, Akaike Info. Crt.=     -0.988 o
   o Autocorrel: Durbin-Watson Statistic =   0.92721,   Rho =      0.53640 o
   =======================================================================¼
     Variable  Coefficient   Standard Error  t-ratio  P[|T|>=t]   Mean of X
    _____________________________________________________________________
       Constant   -5.9462          2.3763       -2.502   0.02439    n.a.
       LRPREG1   -0.99093E-01     0.20475       -0.484   0.63540   1.967
       LRPDIE1    0.42781         0.37329        1.146   0.26974   1.362
       LRGDPCAP    2.0153         0.31869        6.324   0.00001   7.207
```

The coefficients here may be interpreted as elasticities. Thus we have the following:

- the own-price elasticity of demand for regular gasoline is -0.099, which is plausible. However it is not statistically significantly different from zero (the p-value is a high 0.645). This is not a reason for ignoring the estimate, but it does indicate that the elasticity has not been estimated with much precision.
- The cross-price elasticity of demand for regular gasoline, with respect to the price of diesel fuel, is 0.43. This is plausible, although somewhat higher than one might expect; it too is not statistically significant.
- The income elasticity of demand for regular gasoline is 2.02 and statistically significant. This is high, but plausible.

However this is not yet a satisfactory regression, because the low Durbin-Watson Statistic indicates that there is significant autocorrelation. Adding a time trend (results not shown here) removes the autocorrelation, but radically alters some of the coefficients - for instance cutting the income elasticity down to 0.32.

Step 5b: Explore functional form: Box-Cox

One can try to determine whether a linear, or log, or other functional form is appropriate by first transforming the variables using the Box-Cox transformation. This transformation (on variable *q*) is given by

$$q^{(\lambda)} = (q^{\lambda}-1)/\lambda \qquad \text{if } \lambda \neq 0$$

$$= \log(q) \qquad \text{if } \lambda=0.$$

In order to transform the variables one needs to estimate λ, which is not yet known. One can search over a grid for the value of λ which maximizes the likelihood of the function, or one can apply an optimization program to find the value of λ which maximizes the likelihood function; a good statistical package will do this in response to a few relatively simple commands. The results of applying maximum likelihood estimation to the Box-Cox transformed model are as follows:

```
==============================================================================
º  LIMDEP Estimation Results                       Run log line   55  Page  26 º
º  Current sample contains      19 observations.                               º
==============================================================================¼
    +-----------------------------------------------------------------------+
    º Box-Cox Regression -- OLS Starting Values                            º
    º Ordinary    least squares regression    Weighting variable = ONE     º
    º Dependent variable is QREGCAPN  Mean =    8.06539, S.D. =      2.1616 º
    º Model size: Observations =      19, Parameters =   4, Deg.Fr. =    15 º
    º Residuals:  Sum of squares=    19.0725     Std.Dev. =         1.12761 º
    º Fit:        R-squared = 1.27150, Adjusted R-squared =         1.32580 º
    º Diagnostic: Log-L =    -26.9960, Restricted(á=0) Log-L =     -41.0923 º
    º             Amemiya Pr. Crt.=    1.539, Akaike Info. Crt.=      3.263 º
    ======================================================================¼
      Variable  Coefficient   Standard Error  z=b/s.e. P[|T|>=t]   Mean of X
      ____________________________________________________________________
        RPREG1    -0.81910E-01     0.24143       -0.339   0.73441   7.238
        RPDIE1     0.98171         0.80679        1.217   0.22368   3.918
        RGDPCAPN    12.935          1.8203        7.106   0.00000   1.356
        Constant   -12.732          4.2189       -3.018   0.00255    n.a.

==============================================================================
º  LIMDEP Estimation Results                       Run log line   55  Page  27 º
º  Current sample contains      19 observations.                               º
==============================================================================¼
    ======================================================================
    º Box-Cox Nonlinear Regression Model                                   º
    º Maximum likelihood estimator       Heteroscedasticity:W(i) = ONE     º
    º Dependent variable is QREGCAPN  Mean =    8.06539, S.D. =      2.1616 º
    º Model size: Observations =      19, Parameters =   4, Deg.Fr. =    15 º
    º Residuals:  Sum of squares=   0.634081E-01 Std.Dev. =         0.05777 º
    º Fit:        R-squared = 0.99929, Adjusted R-squared =         0.99932 º
    º             Note:  Not using OLS.  R-squared is not bounded in [0,1]  º
    º Model test: F[  3,     15] = 6995.49,    Prob value =         0.00000 º
    º Diagnostic: Log-L =     27.2149, Restricted(á=0) Log-L =     -41.0923 º
    º             Amemiya Pr. Crt.=    0.004, Akaike Info. Crt.=     -2.444 º
    º Transformations: RHS = Lambda  , LHS = Lambda                        º
    º Elasticities have been kept in matrix EPSILON                        º
    º Log-likelihood accounting for the LHS transformation   =    -25.64911 º
    ======================================================================¼
      Variable  Coefficient   Standard Error  z=b/s.e. P[|T|>=t]   Mean of X
      ____________________________________________________________________
        RPREG1    -0.10601         0.17514       -0.605   0.54498   7.238
        RPDIE1     0.31647         0.35926        0.881   0.37838   3.918
        RGDPCAPN    1.0550          1.7776        0.593   0.55286   1.356
        Constant   0.97066         0.45167        2.149   0.03163    n.a.
        Lambda    -0.35143         0.91363       -0.385   0.70050    n.a.
       sigma sq   0.33373E-02     0.12601E-01     0.265   0.79114    n.a.
```

The estimated value of λ is -0.35, and is not statistically different from zero. This suggests that the log-log model (which implicitly assumes λ=0) is better than the linear model (which assumes λ=1); this is not a strong conclusion however, because the estimate of λ is very imprecise. Nonetheless in what follows we will work exclusively with the log-log version of the demand curve.

Step 6: Deal with autocorrelation.

The classical regression model, given by

$$\mathbf{y}_t = \beta'\mathbf{x}_t + \varepsilon_t \qquad (1)$$

assumes that the disturbance term (ε_t) is normally distributed with mean 0 and variance σ^2. With first-order autocorrelation (which is the commonest type) we have

$$\varepsilon_t = \rho.\varepsilon_{t-1} + u_t \qquad (2)$$

where ρ is the (first-order) autocorrelation coefficient and it is assumed that u_t is normally distributed with zero mean and constant variance. The Durbin-Watson statistic tests for the presence of first-order autocorrelation; when it is close to 2 there is no autocorrelation, but if the number approaches 0 or 4 then there is evidence of autocorrelation.

The problem is that if we estimate equation (1) when there is autocorrelation present, the estimates of the coefficients will be biased. The solution is to estimate ρ and to transform the variables to give

$$\mathbf{y}_t - \rho.\mathbf{y}_{t-1} = \beta'(\mathbf{x}_t - \rho\mathbf{x}_{t-1}) + u_t \qquad (3)$$

and then to apply ordinary least squares to this equation. One first needs an estimate of ρ, and there are a number of techniques for getting this; the estimates differ slightly, depending on the technique used. Here are the results of applying a maximum likelihood approach to estimating ρ:

```
===============================================================================¼
º  LIMDEP Estimation Results                      Run log line   26  Page  10 º
º  Current sample contains      19 observations.                              º
===============================================================================¼

                ==============================================
                º AR(1) Model:     e(t) = rho * e(t-1) + u(t) º
                º Initial value of rho       =        0.53640 º
                º Maximum iterations         =             20 º
                º Iter=  8, SS=      0.137, Log-L=  18.989842 º
                º Final value of Rho    =             0.91463 º
                º Durbin-Watson:   e(t) =             0.17864 º
                º Std. Deviation:  e(t) =             0.23645 º
                º Std. Deviation:  u(t) =             0.09559 º
                º Durbin-Watson:   u(t) =             2.14519 º
                º Autocorrelation: u(t) =            -0.07259 º
                º N[0,1] used for significance levels         º
                ==============================================¼

      Variable  Coefficient   Standard Error  z=b/s.e. P[|T|>=t]   Mean of X
      ---------------------------------------------------------------------
        Constant    4.1724          3.4052        1.225   0.22047    n.a.
        LRPREG1   -0.45080         0.31944       -1.411   0.15819   1.967
        LRPDIE1    0.18787         0.29252        0.642   0.52071   1.362
        LRGDPCAP   0.76181         0.44956        1.695   0.09016   7.207
        Rho        0.91463         0.95291E-01    9.598   0.00000    n.a.
```

The estimated value of ρ is high (i.e. close to 1) and highly significant. There is no remaining autocorrelation. The estimated coefficients are all reduced (in absolute terms), when compared to the results of the estimation which did not correct for autocorrelation. This is a viable and plausible model.

Step 7: Exploring dynamics

Most of the equations estimated so far are static, in the sense that they assume that consumers fully adjust their quantity demanded in year *t* in response to the income and price levels observed in year *t*. For goods such as gasoline it is more plausible that consumers adjust with a lag. For instance, a higher price of gasoline may eventually lead consumers to replace large cars with small ones, but this takes time. The *partial adjustment* model begins with

$$y_t = y_{t-1} + k(y^*_t - y_{t-1})$$

where y^*_t is the desired level of y_t and k is the proportion of the adjustment from the previous years level to this year's desired level that takes place in year *t*. The adjustment parameter k is expected to be between 0 (slow adjustment) and 1 (rapid adjustment). If one then assumes that

$$\mathbf{y}^*_t = \beta'\mathbf{x}_t + \varepsilon_t \qquad (4)$$

and rearranges, this yields the equation which has to be estimated:

$$\mathbf{y}_t = k.\beta'\mathbf{x}_t + (1-k)\mathbf{y}_{t-1} + k.\varepsilon_t \qquad (5)$$

This is very like the classical regression equation given in (1), except that there is a lagged dependent variable on the right hand side. The results of estimating equation (5) are as follows:

```
================================================================================
º  LIMDEP Estimation Results                       Run log line   28  Page  11 º
º  Sample was reset:  REJECT   YEAR=1978$                                     º
º  Current sample contains      18 observations.                              º
===============================================================================¼
   ========================================================================
   º Ordinary    least squares regression    Weighting variable = ONE      º
   º Dependent variable is LQREGCAP  Mean =    8.93605, S.D. =      0.2015 º
   º Model size: Observations =      18, Parameters =   5, Deg.Fr. =    13 º
   º Residuals:  Sum of squares=   0.763599E-01 Std.Dev. =         0.07664 º
   º Fit:        R-squared = 0.88938, Adjusted R-squared =         0.85534 º
   º Model test: F[  4,     13] =   26.13,    Prob value =         0.00000 º
   º Diagnostic: Log-L =     23.6231, Restricted(á=0) Log-L =       3.8082 º
   º             Amemiya Pr. Crt.=    0.008, Akaike Info. Crt.=     -2.069 º
   º Autocorrel: Durbin-Watson Statistic =   2.36533,   Rho =     -0.18266 º
   =======================================================================¼
     Variable  Coefficient   Standard Error  t-ratio  P[|T|>=t]   Mean of X
    _____________________________________________________________________
       Constant  -0.65452E-01      1.7097       -0.038   0.97004    n.a.
       LRPREG1   -0.26220         0.13378       -1.960   0.07179   1.975
       LRPDIE1    0.78846E-01     0.23324        0.338   0.74073   1.367
       LRGDPCAP   0.41417         0.33218        1.247   0.23447   7.196
       LQRE[-1]   0.71717         0.14038        5.109   0.00020   8.968
```

This equation fits well (adjusted R^2 = 0.86) and the coefficients are reasonable. Durbin's h statistic should be used instead of the Durbin-Watson test here, because of the presence of the lagged dependent variable, but autocorrelation does not appear to be a problem here. The estimated value of k is 0.28 (= 1-0.72), which implies that 28% of the adjustment to prices and income takes place in a given year. This is a slow reaction, but not implausible. The coefficients on the independent variables may be thought of as short-run elasticities; when divided by the estimate of *k* they yield long-run elasticities. This gives the following:

	Elasticity of demand for regular gasoline per capita	
	short-run	long-run
with respect to the price of regular gasoline	-0.26	-0.93
with respect to the price of diesel fuel	0.08	0.28
with respect to GDP per capita	0.41	1.46

These are certainly reasonable numbers, but note that the estimated coefficients on two of the variables - the price of diesel fuel, and GDP per capita, and not statistically significant.

Step 8: An Error-Correction Model

Until recently, most analysis stopped at Step 7 (e.g. Gately, Greene, etc.). The analyst, using his or her best judgment, would choose a "best" model, and typically base the rest of their discussion on the results of the favored model. Much of the art of applied economics is in determining which model is the most accurate and appropriate representation of reality.

Some authors do a lot of experimentation, but this is not desirable, particularly with a very small time series such as the one considered here. It is simply asking too much of a short series of numbers to pick from a multitude of possible specifications and to come up with reliable estimates. Experimentation tends to lead to over-fitting, and hence to reported results that are not in fact as precise as they seem.

When there are enough observations, in a time series, it is possible to apply even more elaborate statistical techniques, and to build an error-correction model. We illustrate this procedure with the same data set, despite the fact that there are not really enough observations in this case for the procedure to be useful enough to generate believable and useful results.

The first question to ask is whether the different time series are stationary. A stationary series is one that does not show any clear trend over time. The problem here is that when series are not stationary they may seem to be closely related - because they are drifting in the same direction over time - even though there may not in fact be any relationship between them. One might then observe a spurious correlation, and thus draw the wrong inferences from the data. For example, in

Europe the number of people going to church on Sundays has been falling, while GDP has been rising. A regression of GDP against church-going would show a statistically significant negative relationship. But there may be no real causality involved. The commonest solution is to purge the series of the time trend, usually by taking differences; thus instead of regressing $\mathbf{y}_t$ on $\mathbf{x}_t$ one eventually regresses $\mathbf{y}_t$-$\mathbf{y}_{t-1}$ on $\mathbf{x}_t$-$\mathbf{x}_{t-1}$. Of course this step is not necessary if the series are in fact stationary, so one needs to check this first.

To test for stationarity, estimate the equation

$$\Delta X_t = \phi_0 + \phi_1 t + \phi_2 X_{t-1} + \varepsilon_t. \qquad (6)$$

for all the variables (dependent and independent); here $\Delta X_t = X_t - X_{t-1}$. If the estimated value of ϕ_2 is significantly negative, then one rejects the null hypothesis of a "unit root" - i.e. it is reasonable to suppose that the series X_t is stationary. Then one need go no further, and the results found in the preceding steps will apply.

But if the "t-statistic" on ϕ_2 is low, then one cannot reject the unit root hypothesis that the series is in fact non-stationary. The "t-statistic" in this case, calculated by dividing the estimated coefficient by its estimated standard error, has to be compared with the significance tables compiled by Dickey and Fuller. In some cases researchers also include ΔX_{t-2} on the right-hand side, in which case the augmented Dickey-Fuller test is applied.

Suppose that a subset of *n* variables are non-stationary. Then the steps are:

- Apply OLS to the these variables in their levels - i.e. estimate a regression along the lines shown in equation (1). If this equation is statistically significant, and if the residuals are themselves stationary, then one has the *co-integrating vector*, which gives the long-run relationship between the variables. The elasticities from this equation may be treated as long-run elasticities. Call the estimated residuals from this equation e_t..
- Now estimate the relationship in its differenced form, and include the lagged residuals e_{t-1} in this equation. Such an estimating equation might look like the following:

$$\Delta\ln(Y_t) = a + b.\Delta\ln(X_t) + c.\Delta\ln(Y_{t-1}) + d.e_{t-1}$$

(Note that the lagged value of the dependent variable could be omitted.) This equation may be interpreted as yielding short-run elasticities. It should also include on the right hand side any stationary variables (differenced); after all these may have a short-run effect on the dependent variable, even though they cannot have a permanent effect on Y. The estimate of the coefficient *d* indicates how quickly the dependent variable adjusts towards its long-run level. For instance Bentzen and Engsted, in a study of energy consumption in Denmark, estimated *d* to be a statistically significant -0.238; they interpret this as indicating that "in the case we are off the long-run demand curve, energy consumption adjusts towards its long-run level with about one quarter of the adjustment taking place within the first year" (p.13).

We may apply the error-correction approach to the Madagascar gasoline data. First we test for stationarity, estimating equation (6) for all of the variables. Here are the essential results:

```
=========================================================================
º Ordinary    least squares regression    Weighting variable = ONE      º
º Dependent variable is DLQREGCA  Mean =   -0.03159, S.D. =      0.1002 º
º Model size: Observations =      18, Parameters =   3, Deg.Fr. =    15 º
º Residuals:  Sum of squares=   0.109323     Std.Dev. =         0.08537 º
º Fit:        R-squared = 0.35951, Adjusted R-squared =         0.27411 º
º Autocorrel: Durbin-Watson Statistic =   2.35456,   Rho =     -0.17728 º
========================================================================¼
   Variable  Coefficient   Standard Error  t-ratio  P[|T|>=t]   Mean of X
   ______________________________________________________________________
     Constant   -6.1220         15.085       -0.406   0.69060    n.a.
     YEAR        0.38411E-02     0.70035E-02  0.548   0.59145   1988.
     LQRE[-1]  -0.17216         0.15354      -1.121   0.27980   8.968

=========================================================================
º Ordinary    least squares regression    Weighting variable = ONE      º
º Dependent variable is DLRPREG1  Mean =   -0.00853, S.D. =      0.1015 º
º Model size: Observations =      18, Parameters =   3, Deg.Fr. =    15 º
º Residuals:  Sum of squares=   0.905366E-01 Std.Dev. =         0.07769 º
º Fit:        R-squared = 0.48337, Adjusted R-squared =         0.41449 º
º Autocorrel: Durbin-Watson Statistic =   1.95380,   Rho =      0.02310 º
========================================================================¼
   Variable  Coefficient   Standard Error  t-ratio  P[|T|>=t]   Mean of X
   ______________________________________________________________________
     Constant    30.459          8.4310        3.613   0.00256    n.a.
     YEAR      -0.14905E-01     0.41605E-02  -3.582   0.00272   1988.
     LRPR[-1]  -0.42556         0.15058      -2.826   0.01277   1.984

=========================================================================
º Ordinary    least squares regression    Weighting variable = ONE      º
º Dependent variable is DLRPDIE1  Mean =    0.01155, S.D. =      0.1103 º
º Model size: Observations =      18, Parameters =   3, Deg.Fr. =    15 º
º Residuals:  Sum of squares=   0.123399     Std.Dev. =         0.09070 º
º Fit:        R-squared = 0.40308, Adjusted R-squared =         0.32349 º
º Autocorrel: Durbin-Watson Statistic =   1.87743,   Rho =      0.06128 º
========================================================================¼
   Variable  Coefficient   Standard Error  t-ratio  P[|T|>=t]   Mean of X
   ______________________________________________________________________
     Constant  -0.80620          8.1901       -0.098   0.92289    n.a.
     YEAR       0.96927E-03     0.41261E-02   0.235   0.81745   1988.
     LRPD[-1]  -0.81779         0.25702      -3.182   0.00619   1.356

=========================================================================
º Ordinary    least squares regression    Weighting variable = ONE      º
º Dependent variable is DLRGDPCA  Mean =   -0.01714, S.D. =      0.0499 º
º Model size: Observations =      18, Parameters =   3, Deg.Fr. =    15 º
º Residuals:  Sum of squares=   0.329999E-01 Std.Dev. =         0.04690 º
º Fit:        R-squared = 0.21908, Adjusted R-squared =         0.11495 º
º Autocorrel: Durbin-Watson Statistic =   1.43401,   Rho =      0.28299 º
========================================================================¼
   Variable  Coefficient   Standard Error  t-ratio  P[|T|>=t]   Mean of X
   ______________________________________________________________________
     Constant    6.8389          7.3372        0.932   0.36605    n.a.
     YEAR      -0.23446E-02     0.32143E-02  -0.729   0.47698   1988.
     LRGD[-1]  -0.30447         0.16756      -1.817   0.08923   7.213
```

In every case coefficient on the lagged dependent variable is negative, which is to be expected. And in every case it is not statistically significant, because the critical t-value for the Dickey-Fuller test is about 3.5. Thus we cannot reject the unit root hypothesis that all the variables are non-stationary - i.e. they are drifting over time.

Thus we applied OLS to the whole model; the results are given above in step 5a. We used the residuals from this equation to estimate the following error-correction model:

```
===============================================================================
º  LIMDEP Estimation Results                      Run log line   49  Page  22 º
º  Current sample contains      18 observations.                              º
===============================================================================¼
   ========================================================================
   º Ordinary    least squares regression    Weighting variable = ONE      º
   º Dependent variable is DLQREGCA  Mean =   -0.03159, S.D. =      0.1002 º
   º Model size: Observations =      18, Parameters =   5, Deg.Fr. =    13 º
   º Residuals:  Sum of squares=   0.782831E-01 Std.Dev. =         0.07760 º
   º Fit:        R-squared = 0.54136, Adjusted R-squared =         0.40024 º
   º Model test: F[  4,     13] =    3.84,    Prob value =         0.02848 º
   º Diagnostic: Log-L =     23.3993, Restricted(á=0) Log-L =      16.3838 º
   º             Amemiya Pr. Crt.=    0.008, Akaike Info. Crt.=     -2.044 º
   º Autocorrel: Durbin-Watson Statistic =   1.73287,   Rho =      0.13357 º
   ========================================================================¼
     Variable  Coefficient   Standard Error  t-ratio  P[|T|>=t]   Mean of X
     ____________________________________________________________________
       Constant  -0.33262E-01     0.20794E-01   -1.600   0.13369    n.a.
     DLRPREG1  -0.52943         0.29732       -1.781   0.09833 -0.8528E-02
     DLRPDIE1   0.26088         0.25240        1.034   0.32018  0.1155E-01
     DLRGDPCA   0.47736         0.42336        1.128   0.27990 -0.1714E-01
     ELQR[-1]  -0.38071         0.16041       -2.373   0.03373 -0.6099E-02
```

This indicates that if demand has been pushed off its equilibrium level, about 38% of the disequilibrium will be corrected for in any given years. The short-run elasticities are reasonable, although perhaps a bit too high for the own-price elasticity of demand. It is a pity that this regression is barely statistically significant; only the error-correction term can really be said to have a significant influence.

In Sum

The estimated elasticities from the above estimations are summarized in Table 1. Unfortunately they differ substantially, depending on the method used. The straightforward linear and log-log models are not satisfactory, because there is autocorrelation present. The error correction model may be stretching the limited data too far. The partial adjustment model yields plausible results, and gives both short-run (one year) and long-run elasticities; these are the ones I consider to be most satisfactory in this case. The AR1 estimates lie between the lower and upper bounds of the partial adjustment estimates, and might perhaps be thought of as some sort of average of the short-run and long-run elasticities.

Table 1
Estimated elasticities of demand for the quantity of regular gasoline in Madagascar

	Estimated elasticity of demand for regular gasoline,

Model	with respect to: the real price of regular gasoline		the real price of diesel fuel		real GDP/capita	
	short-run	*long-run*	*short-run*	*long-run*	*short-run*	*long-run*
Linear model, no time trend, at mean	-0.07		0.48		2.18*	
Linear model, time trend, at mean	-1.07*		1.12*		0.51	
Log-log model, no time trend	-0.10		0.43		2.02*	
Log-log model, time trend	-0.98*		093*		0.32	
AR1 model	-0.45		0.19		0.76*	
Partial adjustment model	**-0.26***	**-0.93***	**0.08**	**0.28**	**0.41**	**1.46**
Error correction model	-0.53	-0.10	0.26	0.43	0.48	2.02
Memo items:						
Means of independent variables	7.238		3.918		1356.4	
Coefficients, linear model, no trend	-81.91		81.71		12.94	
Coefficients, linear model, trend	-1193.4*		2100.5*		3.006	

Notes: * denotes statistically significant at 10% level or better. Mean of dependent variable is 8065.4.

Some further considerations

This example has focused on the determinants of demand for regular gasoline. But about 5% of the gasoline sold in Madagascar is premium gasoline, and is presumably a close substitute for regular gasoline. One could add the real price of premium gasoline to the regression, although this would use up one valuable degree of freedom. Alternatively one could estimate the demand for all gasoline, adding together the amounts of regular and premium gasoline sold. In this case one would have to construct a gasoline price index - a weighted average of the price of regular and of premium gasoline.

An exercise similar to this one would be worth applying to the demand for diesel fuel, which is two and a half times larger (now) than the demand for gasoline.

The emphasis in this note is on the practice of applied econometrics. One could push the analysis further. For instance one could check for the presence of heteroskedasticity, although this is not usually a serious problem in time-series analysis of this nature. Or one could try other forms of lag structure, or other variables. And one might want to add dummy variables for those years in which there was an exceptionally disruptive event (e.g. a coup, an oil price shock, etc.). But with the exception of such dummy variables, it is rare that such refinements add much to the simpler, and generally more robust, analysis outlined here.

For the estimation of demand curves for tobacco products, and for alcoholic beverages, the lag structure may be less important than in the case of petroleum products. Presumably people adjust fairly quickly to changes in the price of alcohol or tobacco; this is not guaranteed however, because alcohol and (especially) tobacco have addictive properties and people may have to struggle to give up their consumption, even when it becomes too expensive.

Perhaps the biggest challenge in modeling the demand for excisable tobacco and alcohol products is in finding a viable price series for informal substitutes. Yet the prices of locally-made tobacco, and of artisanal alcohol, are likely to influence strongly the demand for these products in the formal sector.

3. Elasticities in comparative perspective

Petroleum Products

A sampling of estimates for the elasticity of demand for gasoline (in total, or per capita, or per vehicle, or the demand for miles driven) is given in Table 2. An enormous number of studies of gasoline (or fuel or energy) demand have been undertaken for the developed countries (see Dahl, and also Dahl and Sterner, for summaries), but very few for less-developed countries. There is a gap here which needs to be filled. The key findings for developed countries are

- the own-price elasticity of demand is (absolutely) very low, both in the short-run and even in the long-run. The implication here is that the revenue-maximizing tax on motor fuel is likely to be high (see methodological note No. 3).
- the income elasticity of demand is less than one, and so taxes on motor fuel are not likely to be very income elastic (i.e. will not rise as quickly as GDP; see methodological note No. 1).

However I would expect the income elasticity to be greater than one in most less-developed countries; as income rises in LDCs, many people acquire motorbikes and cars and so within a certain income range the consumption of motor fuel almost certainly rises more quickly than income.

Table 2
Demand elasticities for petroleum products

	Own-price elasticity of demand	Income or GDP elasticity of demand

	short-run	long-run	short-run	long-run
Madagascar, annual data, 1978-1996, partial adjustment (from Table 1)				
Dependent variable: gasoline per capita	-0.26	-0.93	0.41	1.46
11 Asian countries, annual data 1973-87, pooled. (McRae)				
Dependent variable: gasoline per vehicle	-0.13	-0.26	0.6	0.66
Dependent variable: vehicles per person	-0.7	-0.31		
OECD countries, ... (Baltagi and Griffin)				
Dependent variable: gasoline/vehicle		-0.55		0.54
Dependent variable: vehicles/person		-0.66		
US, annual data, 1966-1989, partial adjustment. (Greene)				
Dependent variable: vehicle miles traveled*	-0.12	-0.33	-0.07	-0.20
US, annual data, 1966-88, log-log model. (Gately)				
Dependent variable: Vehicle miles travelled*	-0.09		0.52	
US, state data, 1970-1991, partial adjustment. (Haughton & Sarkar)				
Dependent variable: gasoline	-0.14	-0.30		
Various industrial countries, since about 1970. (Dahl and Sterner)				
Dependent variable: gasoline per capita	≈-0.27	≈-0.84	≈0.44	≈1.33
Denmark, annual data, 1948-1991, error correction model (Bentzen)				
Dependent variable: gasoline per capita	-0.32	-0.41	0.89	1.04

Notes: * From regression which includes number of drivers on right hand side.

Cigarettes and tobacco

A sampling of demand elasticities for cigarettes and tobacco is shown in Table 3. The evidence from the US indicates that in the short-run (i.e. about one year) the demand for cigarettes is quite inelastic - i.e. even if the price were rise 10%, the quantity demanded would fall by substantially less than 10%, in fact by somewhere between 2% and 4%. On the other hand the long-run elasticities are more substantial. The "addiction models" of cigarette demand are based on the idea that consumption now depends on how much one consumed in the past and how much one will consume in the future; the addictive qualities of cigarettes make it harder for smokers to react quickly to a change in the price of cigarettes. In these cases the demand equations are estimated with lagged dependent variables on the right hand side (the "myopic addiction" model) or with both lagged and lead dependent variables on the right hand side (the "rational addiction" model). There is a good discussion in Keeler et al.

Almost no studies of this nature have been reported for less-developed countries. One exception is Chapman and Richardson's results for Papua New Guinea. They did not have information about the retail price of cigarettes or tobacco, so they measured the response of demand to a change in the (real) excise taxes on tobacco and on cigarettes. Their excise elasticities are quite high (in absolute terms), and understate the price elasticities.

Table 3
Demand elasticities for cigarettes and tobacco

	Own-price elasticity of

	demand	
	short-run	long-run
USA		
Cigarettes. Becker, Grossman & Murphy, 1994. State data over time.	-0.36	-0.79
Cigarettes. Chaloupka, 1991. Individual data.	-0.20	-0.45
Cigarettes. Keeler et al, 19xx. California monthly data, 1980-90.		
myopic addiction (i.e. with lagged variables)	-0.34	-0.47
rational addiction (i.e. with lead and lagged variables)	-0.36	-0.58
no addictive behavior: with time trend	-0.20	
no addictive behavior: without time trend	-0.46	
	w.r.t. price of cigarettes	w.r.t. price of tobacco
Papua New Guinea.		
Chapman and Richardson. Annual data, 1973-86		
Cigarettes. Excise elasticities.*	-0.71	0.50
Tobacco Excise elasticities.*	0.62	-0.50

Note: * Excise elasticities give % change in quantity demanded ÷ % change in excise tax rate. These understate the price elasticities of demand.

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Appendix 1
Data for Madagascar Petroleum Product Demand Estimations

	Vol, MT		Val, pre T, '000 FMG		TUPP paid, '000 FMG		TUPP/liter		
	Regular	Diesel	Regular	Diesel	Regular	Diesel	Regular	Diesel	GDP
	qreg	qdie	vreg	vdie			treg	tdie	gdp
1978	111,813	159,146	10,076,699	8,222,596	2,560,518	87,643	22.90	8.50	602.4
1979	93,973	183,492	8,468,095	9,468,132	2,236,557	121,287	23.35	6.67	736.8
1980	107,997	184,878	17,141,575	15,623,300	2,994,757	61,291	24.27	7.24	854.0
1981	92,547	169,865	24,926,239	21,528,860	6,145,121	335,939	55.88	21.23	976.8
1982	77,671	149,055	25,043,849	22,027,199	4,388,412	310,343	56.20	21.65	1,233.2
1983	76,454	147,354	24,651,445	21,775,827	4,319,651	332,896	32.06	18.39	1,511.5
1984	70,867	146,541	22,758,985	22,784,373	3,270,720	3,204,948	46.15	21.87	1,695.0
1985	72,672	150,617	25,962,544	26,071,863	1,795,489	1,941,754	24.71	12.89	1,893.2
1986	75,260	167,900	26,170,620	28,052,604	2,414,353	3,008,060	32.08	17.92	2,203.8
1987	73,396	165,902	33,262,627	36,274,638	2,833,526	3,890,236	38.61	23.45	2,743.2
1988	69,107	163,649	43,294,153	50,939,024	3,145,751	4,210,689	45.52	25.73	3,436.8
1989	72,488	178,934	48,781,202	60,950,338	3,555,134	4,420,217	49.04	24.70	4,005.3
1990	75,788	193,188	52,445,296	67,680,196	2,273,640	2,897,820	30.00	15.00	4,603.9
1991	67,448	194,599	51,339,169	80,742,368	4,384,120	6,324,468	65.00	32.50	4,913.6
1992	74,002	217,107	42,433,865	99,742,543	19,487,141	7,689,204	263.33	35.42	5,593.1
1993	82,658	219,442	36,720,817	102,168,475	32,925,437	22,584,225	398.33	102.92	6,450.9
1994	87,338	250,550	46,485,396	126,760,521	48,225,569	39,733,850	536.67	156.67	9,131.2
1995	95,893	260,822	90,642,072	197,744,385	56,949,125	37,987,703	595.00	192.08	13,639.9
1996	103,503	257,906	110,336,800	220,877,142	71,783,760	70,043,100	690.00	345.00	16,403.7

			Pump P FMG/l		Real prices/l		Qty per cap		Pre-tax real prices	
	CPI	Pop	Regular	Diesel	Regular	Diesel	Regular	Diesel	Regular	Diesel
	cpi	pop	preg1	pdie1	rpreg1	rpdie1	qregcap	qdiecap	ptrpreg1	ptrpdie1
1978	14.6	8.2	90.4	51.8	6.19	3.54	13,570	19,314	4.62	2.96
1979	16.7	8.5	116.3	63.2	6.96	3.79	11,095	21,664	5.56	3.39
1980	19.7	8.8	153.3	81.4	7.78	4.13	12,300	21,057	6.55	3.76
1981	25.7	9.0	248.0	118.8	9.65	4.62	10,329	18,958	7.48	3.80
1982	33.9	9.3	303.3	140.2	8.95	4.13	8,316	15,959	7.29	3.50
1983	40.5	9.4	323.0	148.0	7.98	3.65	8,133	15,676	7.18	3.20
1984	44.4	9.71	360.9	172.2	8.13	3.88	7,298	15,092	7.09	3.39
1985	49.1	9.98	382.0	186.0	7.78	3.79	7,282	15,092	7.28	3.53
1986	56.3	10.1	382.0	186.0	6.79	3.30	7,451	16,624	6.22	2.99
1987	64.7	10.37	475.3	233.7	7.35	3.61	7,078	15,998	6.75	3.25
1988	82.1	10.63	673.8	337.9	8.21	4.12	6,501	15,395	7.65	3.80
1989	89.5	10.91	672.0	337.0	7.51	3.77	6,644	16,401	6.96	3.49
1990	100.0	11.2	719.3	363.8	7.19	3.64	6,767	17,249	6.89	3.49
1991	108.5	11.49	826.1	446.9	7.61	4.12	5,870	16,936	7.01	3.82
1992	124.4	12.08	838.0	494.7	6.74	3.98	6,126	17,972	4.62	3.69
1993	136.8	12.42	844.1	570.2	6.17	4.17	6,655	17,668	3.26	3.42
1994	190.1	12.77	1078.5	662.4	5.67	3.48	6,839	19,620	2.85	2.66
1995	276.0	13.13	1536.0	1199.8	5.57	4.35	7,303	19,865	3.41	3.65
1996	331.1	13.47	1757.4	1444.8	5.31	4.36	7,684	19,147	3.22	3.32
					rpreg1	rpdie1	qregcap	qdiecap	ptrpreg1	ptrpdie1
				Means	7.24	3.92	8,065	17,668	5.89	3.43